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DISTRIBUTION OF TURBULENCE CHARACTERISTICS IN A CHANNEL WITH INTENSIVE INJECTION

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A large number of studies (see, e.g., [1]) have been devoted to aspects of the distribution of flow characteristics in channels with injection. A theoretical analysis of the corresponding solution of the Navier–Stokes equations for laminar flow was first made in [2]. Subsequent experimental studies [3-7] showed that with a turbulent flow regime, the profiles of the longitudinal and transverse components of the velocity vector are described well by limit relations (infinitely large Reynolds number for injection) in [2]. This result, evidence of the high degree of stability of the flow, can be attributed to laminarization of the flow as it is accelerated due to distribution of the injection in the channel [8]. Use of the Prandtl model to describe the distribution of the turbulence characteristics in a channel with injection [9] leads to relations which are inconsistent with this fact.

Here we attempt to construct an approximate semiempirical theory to describe flow characteristics based on the $(k - \varepsilon)$-model of turbulence. By numerically integrating the hydrodynamic equations with the $(k - \varepsilon)$-model, we calculated flow parameters in a broad range of injection Reynolds numbers. The results of the calculations agree well with the experimental data.

1. We are examining a steady flow of a viscous incompressible fluid in a plane channel (Fig. 1) at a sufficiently large distance from the impermeable left wall. Fluid of the density ρ^0 is injected through the permeable top wall of the channel at a constant velocity $q_b{}^0$. The equations describing the flow and the boundary conditions appear as follows in dimensionless form

$$w\frac{\partial w}{\partial z}+v\frac{\partial w}{\partial y}=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial z}\left(\frac{1}{\mathrm{Re}}\frac{\partial w}{\partial z}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mathrm{Re}}\frac{\partial w}{\partial y}\right), \tag{1.1}$$
$$w\frac{\partial v}{\partial z}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial z}\left(\frac{1}{\mathrm{Re}}\frac{\partial v}{\partial z}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mathrm{Re}}\frac{\partial v}{\partial y}\right),\ \frac{\partial w}{\partial z}+\frac{\partial v}{\partial y}=0,$$

where w and v are averaged values of the components of the velocity vector $\mathbf{q}$ along the axes z and y (see Fig. 1):

$$y=0:\ v=0=\partial w/\partial y;\ y=1:\ v=-1,\ w=0;\ z=0:\ w=v=0. \tag{1.2}$$

No conditions are imposed on the right boundary because we are studying a self-similar solution of system (1.1). We use the following as the scales of length, velocity, and pressure in (1.1) and (1.2): h^0 is half the width of the channel; $q_b{}^0$ and $\rho^0 q_b{}^{0 2}$, $\mathrm{Re} = \rho^0 q_b{}^0 h^0/\mu^0$ is the characteristic injection Reynolds number for the problem; μ^0 is the viscosity of the fluid ($\mu^0 = \mu_\ell{}^0 + \mu_t{}^0$, $\mu_\ell{}^0$ and $\mu_t{}^0$ are the laminar and turbulent components).

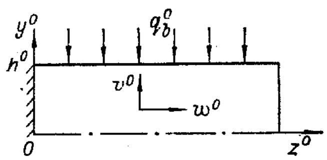

Fig. 1

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It is known [2] that in the limiting case Re → ∞ the solution of problem (1.1), (1.2) has the form

$$w=\frac{\pi}{2}z\cos\frac{\pi}{2}y,\quad v=-\sin\frac{\pi}{2}y,\qquad (1.3)$$
$$p=p_0-\left(\frac{\pi^2}{4}z^2+v^2\right)/2.$$

In fact, as was shown in [10], at Re ≥ 100 (intensive injection) the solution (1.3) satisfactorily approximates the exact solution of the problem (1.1), (1.2). This fact was confirmed experimentally in [3-7].

We will pose the problem of the distribution of turbulence characteristics in the flow at Re → ∞. The solution in this case will be based on the use of the (k − ε)-model of turbulence [11] and Eqs. (1.3). The general transport equation in this case is written as follows in dimensionless form:

$$\frac{\partial}{\partial z}(w\varphi)+\frac{\partial}{\partial y}(v\varphi)-\left[\frac{\partial}{\partial z}\left(\Gamma_\varphi\frac{\partial\varphi}{\partial z}\right)+\frac{\partial}{\partial y}\left(\Gamma_\varphi\frac{\partial\varphi}{\partial y}\right)\right]=S_\varphi,\qquad (1.4)$$

where

$$S_\varphi=\begin{cases}G-\varepsilon, & \varphi\equiv k,\\ c_1G\dfrac{\varepsilon}{k}-c_2\dfrac{\varepsilon^2}{k}, & \varphi\equiv\varepsilon,\end{cases}\qquad (1.5)$$
$$G=c_\mu\frac{k^2}{\varepsilon}\left\{2\left[\left(\frac{\partial w}{\partial z}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2\right\},$$
$$\Gamma_\varphi=c_\mu k^2/(\varepsilon\sigma_\varphi),\ \sigma_k=1,\ \sigma_\varepsilon=1.3.$$

We chose q_b^{02} and h^0/q_b^{02} as the scales of the kinetic turbulence energy k and the rate of its dissipation ε. We took standard values for the constants: $c_\mu = 0.09$, $c_1 = 1.44$, $c_2 = 1.92$. Assuming that the convective terms in system (1.4) dominate the diffusive terms near the wall – through which intensive injection is taking place – and that the characteristics change much more rapidly along a normal to the wall than along the wall ($\partial\varphi/\partial z \ll \partial\varphi/\partial y$), then after some simple transformations we obtain the following from system (1.4)

$$\frac{k^2}{\varepsilon v}\frac{\partial}{\partial y}(kv)=c_\mu\left(\frac{k^2}{\varepsilon}\right)^2\frac{g}{v}-\frac{k^2}{v},\qquad (1.6)$$
$$\frac{k^3}{\varepsilon^2 v}\frac{\partial}{\partial y}(\varepsilon v)=c_1c_\mu\left(\frac{k^2}{\varepsilon}\right)^2\frac{g}{v}-c_2\frac{k^2}{v}\quad (g=\varepsilon G/(c_\mu k^2)).$$

Subtracting the second equation of system (1.6) from the first equation of same and rearranging the terms, we arrive at the relation

$$(kv)^{c_2}\frac{\partial}{\partial y}\left[\frac{\varepsilon v}{(kv)^{c_2}}\right]=-ckvf;\qquad (1.7)$$

$$c=c_\mu(c_2-c_1),\ f=g/v.\qquad (1.8)$$

Equation (1.7), determining the relationship between the variables k and ε with known functions $v = v(z, y)$ and $f = f(z, y)$, can be regarded as a nonlinear differential equation in partial derivatives if we have information on the behavior of any of the variables. Unfortunately, such information is lacking in analytical form in the present case. However, it can be obtained as follows in a first approximation. Taking advantage of a certain degree of arbitrariness allowed in the selection of the constants of the turbulence model when describing specific classes of flows, we put $c_2 = 1$. This allows us to change over from (1.7) to the model equation

$$\frac{\partial}{\partial y}\left(\frac{\varepsilon}{k}\right)=-cf,\qquad (1.9)$$

where the quantity c has been corrected for the new value c_2 in accordance with the first equation of (1.8). Equation (1.9) can be formally integrated over y, which leads to

$$\varepsilon = ckF(z, y);\tag{1.10}$$

$$F(z, y) = \int_y^1 f(z, y_1)\, dy_1.\tag{1.11}$$

In the integration, we assumed that $(\varepsilon/k) = 0$ and $y = 1$. Now we can easily use the first equation of system (1.6) to obtain the relation $\frac{\partial}{\partial y}\left[\ln\left(\frac{kv}{F^{-c_\mu/c}}\right)\right] = -cF/v$. Integration of the latter leads to an expression for the kinetic turbulence energy

$$k = \alpha(z)\frac{F^{-c_\mu/c}}{v}\exp(-c\Phi), \quad \Phi = \int_y^1 \frac{F(z, y_1)}{v(z, y_1)}\, dy_1\tag{1.12}$$

[$\alpha(z)$ is an arbitrary integration function].

With allowance for (1.12), instead of (1.10) we write

$$\varepsilon = -c\alpha(z)\frac{F^{c_\mu/c+1}}{v}\exp(-c\Phi).\tag{1.13}$$

It should be noted that the expression for the function $\alpha = \alpha(z)$ cannot be found within the framework of the given theory. However, in principle $\alpha(z)$ can be determined by correlating the theory with well-known experimental data.

2. We will use solution (1.3) to obtain the distribution of k and ε in the flow. Evaluation of the terms in the expression for G in (1.5) leads to the form of the function g: $g \approx (\partial w/\partial y)^2$. Thus, $f = [-(\pi/2)^4]\cdot z^2 \sin(\pi/2)y$. By integrating this expression, we obtain the following from (1.11):

$$F = \left(\frac{\pi}{2}\right)^3 z^2 \cos\frac{\pi}{2}y.\tag{2.1}$$

Further integration leads to

$$\Phi = \ln\left(\sin\frac{\pi}{2}y\right)^{\beta(z)}.\tag{2.2}$$

This means that instead of Eqs. (1.12) and (1.13), with allowance for (2.1) and (2.2) we have

$$k = -\alpha(z)\left(\frac{\pi}{2}\right)^2 W^4 u^2 V^{\gamma(z)};\tag{2.3}$$

$$\varepsilon = c\alpha(z)\left(\frac{\pi}{2}\right)^3 W^6 u^3 V^{\gamma(z)}.\tag{2.4}$$

Moreover, with allowance for these relations, we obtain the following for the eddy viscosity $\nu_t = c_\mu k^2/\varepsilon$

$$\nu_t = \frac{c_\mu}{c}\alpha(z)\frac{\pi}{2}W^2 u V^{\gamma(z)}.\tag{2.5}$$

Here $W = (\pi/2)z$ is the maximum longitudinal velocity in the channel section; $u = \cos(\pi/2)y$ is the profile of longitudinal velocity; $V = |v| = \sin(\pi/2)y$ is the modulus of the profile of transverse velocity: $\beta(z) = W^2$; $\gamma(z) = -(cW^2 + 1)$.

In accordance with one of the assumptions, $W \gg 1$. Considering that we took $c_2 = 1$ as the model value, we change the sign of the coefficient c, having returned to the initial value $c_2 = 1.92$. This correction hardly changes the value of the coefficient itself, $c = c_\mu(c_2 - c_1)$. Then, instead of (2.3)-(2.5) we write

$$k = -\alpha(z)\left(\frac{\pi}{2}\right)^2 W^4 u^2 V^{\delta(z)};\tag{2.6}$$

$$\varepsilon = c\alpha(z)\left(\frac{\pi}{2}\right)^3 W^6 u^3 V^{\delta(z)};\tag{2.7}$$

$$\nu_t = \frac{c_\mu}{c}\alpha(z)\frac{\pi}{2}W^2 u V^{\delta(z)}, \quad \delta(z) = c(W^2 - 1).\tag{2.8}$$

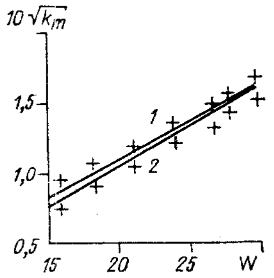

Fig. 2

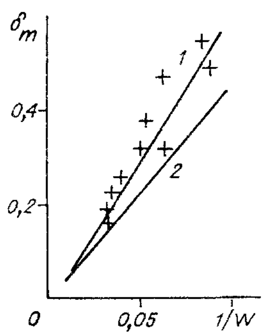

Fig. 3

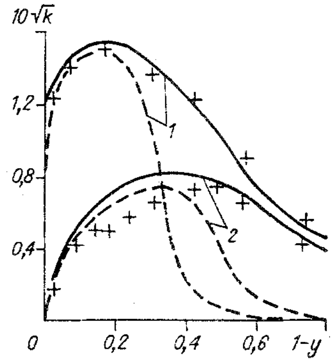

Fig. 4

To determine the relation $\alpha = \alpha(z)$, we use the most complete experimental data available [7, 8]. We will examine the maximum dimensionless level of velocity fluctuations, determined from Eq. (2.6):

$$k_m^{1/2} = \frac{\pi}{2} W^2 \sqrt{-\alpha(z)}\, u(y_m)\, V^{\delta(z)/2}, \tag{2.9}$$

where y_m is the distance from the symmetry plane of the channel to the extremum of the function $k^{1/2}(y, z)$.

Since it follows from the condition of the extremum $(\partial k^{1/2}/\partial y)_{y=y_m} = 0$ that $y_m = (2/\pi)\arctan[\sqrt{(cW^2 - 1)/2}]$ or, with allowance for $W \gg 1$, that

$$y_m \approx \frac{2}{\pi} \operatorname{arctg}\left(\sqrt{c/2}\, W\right), \tag{2.10}$$

then, by considering (2.10), we obtain the following simplified expression for $k_m^{1/2}$ instead of (2.9):

$$k_m^{1/2} = \frac{\pi}{2} W \sqrt{-2\alpha(z)/c}. \tag{2.11}$$

Analysis of the experimental data in [7, 8] shows that

$$k_m^{1/2} \approx 0.05W. \tag{2.12}$$

Comparison of Eqs. (2.11) and (2.12) leads to $\alpha = \text{const} = (-2c/\pi^2)\cdot 10^{-2}$.

Figures 2 and 3 show results of calculation of the relations $k_m^{1/2} = f_1(W)$ and $\delta_m = f_2(W)$ (solid lines) with the use of the value obtained for α (lines 2). Here, $\delta_m = 1 - y_m$, while the plus signs show experimental results. It is evident that there is satisfactory agreement between the theoretical relation and the experimental data for $W > 30$.

In Fig. 4 we constructed theoretical profiles of turbulence energy (dashed lines) in two channel sections: $z = 19.3$ and 10 (lines 1 and 2). The plus signs show data from [8]. It is evident from the graphs that the theoretical curves conform satisfactorily to the experimental results near the channel wall ($y = 1$) up to the extremum of the quantity $k^{1/2}$. At $y \to 0$, the results differ appreciably. This difference is evidently attributable to the coarseness of the theoretical model: diffusion is not considered, the longitudinal gradients of the variables are ignored compared to the transverse gradients, etc. Nevertheless, the model qualitatively describes the phenomenon of flow laminarization in a channel with injection [8] and the evolution of the turbulence energy profile downflow: the maximum of k becomes more pronounced and is displaced toward the channel wall.

3. We performed numerical calculations of flow parameters by the method employed in [12]. The hydrodynamic equations in variables of the stream function – vorticity together with the equations of the standard $(k - \varepsilon)$-model of turbulence [11] – were integrated numerically with the corresponding boundary conditions in a rectangular region (see Fig. 1) of nonuniform grids ranging from 31 × 21 to 51 × 31. The calculations were performed on a BÉSM-6 computer. The characteristic Reynolds number was changed within the range $100 \le \text{Re} \le 3000$. We found that the process of calculating k and ε was unstable in relatively long channels. A possible reason for this is the fact that the model in [11] was intended for

the description of fully developed turbulent flows. Thus, we henceforth used a modified form of the ($k - \varepsilon$)-model from [13] which allowed us to consider possible laminarization of the flow as a result of its acceleration due to the distributed mass supply from the channel walls. In this case, the calculations were stable. The profiles of the calculated velocity-vector components agree well with Eqs. (1.3). Lines 1 in Figs. 2 and 3 show the computational relations $k_m^{1/2} = k_m^{1/2}(W)$ and $\delta_m = \delta_m(W)$ for a comparison with the theoretical and experimental results. The computed profiles $k^{1/2} = k^{1/2}(y)$ are shown in Fig. 4 (solid lines). It can be seen from the graphs that the calculated data agree better with the experimental data than does the theoretical data. Nevertheless, it is evident that despite the limitations of the proposed theory, the derived analytical relations (2.6)-(2.8) satisfactorily describe the distribution of turbulence characteristics in the flow region near the wall and can be used to construct hydrodynamic models of actual processes such as were examined in [7].

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